Short-baseline neutrino oscillations and $(\beta\beta)_{0\nu}$-decay in schemes with an inverted mass spectrum

S.M. Bilenky

Joint Institute for Nuclear Research, Dubna, Russia,

and

Institut für Theoretische Physik, Universität Wien, Boltzmanngasse 5, A-1090 Vienna, Austria.

C. Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino,

Via P. Giuria 1, I-10125 Torino, Italy.

C.W. Kim

Department of Physics and Astronomy, The Johns Hopkins University,

Baltimore, Maryland 21218, USA,

and

Department of Physics, Korea Advanced Institute of Science and Technology, Daeduk, Korea.

S.T. Petcov

Scuola Internazionale Superiore di Studi Avanzati, I-34013 Trieste, Italy,

and

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences,

BG-1784 Sofia, Bulgaria.

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Abstract

We have considered short-baseline neutrino oscillations, ^{3}H β-decay and $(\beta\beta)_{0\nu}$-decay in two schemes with an inverted mass spectrum and mixing of three and four massive neutrino fields. We have analyzed the results of the latest experiments on the search for oscillations of terrestrial neutrinos and we have discussed the compatibility of the LSND indication in favor of neutrino oscillations with the results of the other experiments. In the framework of the models under consideration, it is shown that the observation of $(\beta\beta)_{0\nu}$-decay

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could allow to obtain information about the CP violation in the lepton sector.

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I. INTRODUCTION

The problem of neutrino mass and mixing is the central issue of today's neutrino physics. The study of this problem is associated with the search of new scales in physics. The investigation of the neutrino mass problem is also connected with the hopes to understand the nature of the dark matter in the universe.

At present there are several indications in favor of non-zero neutrino mass and mixing. One indication comes from the solar neutrino experiments. In all four solar neutrino experiments (Homestake [1], Kamiokande [2], GALLEX [3] and SAGE [4]) the observed event rates are significantly lower than the event rates predicted by the Standard Solar Models (see Refs. [5–7]). After the test of the GALLEX and SAGE detectors in special experiments with radioactive ^{51}Cr sources [8,9] the indications in favor of neutrino oscillations coming from solar neutrino experiments have become more significant. Furthermore, a phenomenological analysis of the solar neutrino data based on the assumption that there is no neutrino mixing indicates [10] that the signals due to the ^{7}Be neutrinos in the Cl-Ar and Ga-Ge experiments are substantially lower than the signals predicted by the Standard Solar Models. No plausible astrophysical or nuclear physics explanation of this discrepancy has been proposed so far.

The solar neutrino data can be explained by assuming neutrino mixing with the MSW effect [11,12] or vacuum oscillations [13–15]. In the simplest two-generation scheme, with transitions between ν_e and ν_μ or ν_e and ν_τ, a fit of the data with the MSW mechanism yields the following values for the mixing parameters [16]: $\Delta m^2 \simeq 5 \times 10^{-6}\,\mathrm{eV}^2$ and $\sin^2 2\theta \simeq 7 \times 10^{-3}$ or $\Delta m^2 \simeq 2 \times 10^{-5}\,\mathrm{eV}^2$ and $\sin^2 2\theta \simeq 0.8$, where $\Delta m^2 \equiv m_2^2 - m_1^2$ (m_1 and m_2 are the neutrino masses) and θ is the mixing angle. In the case of vacuum oscillations, a fit of the data yields [17] $\Delta m^2 \simeq 6 \times 10^{-11}\,\mathrm{eV}^2$ and $\sin^2 2\theta \simeq 0.9$.

Another indication in favor of neutrino mixing comes from the results of the experiments on the detection of atmospheric neutrinos [18–20]. In the Kamiokande, IMB and Soudan experiments the ratio of μ-like and e-like events is less than expected. The data can be explained by $\nu_\mu \leftrightarrows \nu_\tau$ or $\nu_\mu \leftrightarrows \nu_e$ oscillations with $\Delta m^2 \simeq 10^{-2}\,\mathrm{eV}^2$ and a large mixing angle.

A third indication in favor of neutrino mixing comes from the possible evidence of $\bar\nu_\mu \to \bar\nu_e$ transitions reported recently by the LSND collaboration [21] (see, however, also Ref. [22]). The explanation of the LSND data in terms of $\bar\nu_\mu \leftrightarrows \bar\nu_e$ oscillations requires a value of Δm^2 of the order of a few eV^2.

Finally, neutrinos are very plausible candidates for the dark matter in the universe if their masses lie in the eV region (see, for example, Refs. [23,24]).

All these indications in favor of neutrino mass and mixing could imply that there are at least three different scales for Δm^2. In this case, the number of massive neutrinos must be more than three. Let us notice that the results of the LEP experiments which proved that the number of neutrino flavors is equal to three (see Ref. [25]) do not constrain the number of massive light neutrinos. In the case of a Dirac neutrino mass term, the total lepton number is conserved, massive neutrinos are Dirac particles and the number of massive neutrinos can be equal to three. In the case of a Dirac and Majorana neutrino mass term, which is typical for GUT models, the total lepton number is not conserved, massive neutrinos are Majorana particles and their number is larger than three.

We will consider here short-baseline oscillations of the terrestrial neutrinos in two possible schemes, one with mixing of three massive neutrino fields and the other with mixing of four massive neutrino fields. In the case of three neutrinos we will assume that the neutrino masses satisfy the relation $m_1 \ll m_2 \simeq m_3$. In the case of four neutrinos we will assume that $m_1 \simeq m_2 \ll m_3 \simeq m_4$. Models with three and four massive neutrinos with this types of mass relations have been considered in the recent articles [26–28] and [29–34] respectively. They are inspired by the experimental indications in favor of neutrino mixing and by astrophysical arguments [32] in favor of the existence of two practically degenerate neutrinos with masses in the eV range. These schemes are compatible with the constraints that follow from the r-process production of heavy elements in the neutrino-heated ejecta of supernovae [35].

We will derive now the general formulas for neutrino oscillations in the framework of the schemes under consideration, that will be used in the subsequent analyses. Let us consider neutrino oscillations in the case of two groups of neutrinos with close masses. We will assume that the masses of the second group are much larger than the masses of the first group and only one squared-mass difference is relevant for oscillations of the terrestrial neutrinos.

According to a general theory of neutrino mixing (see, for example, Refs. [14,36,24]), the left-handed flavor neutrino fields $\nu_{\alpha L}$ are superpositions of the left-handed components of (Dirac or Majorana) massive neutrino fields ν_i:

$$\nu_{\alpha L} = \sum_i U_{\alpha i} \nu_{iL} \quad , \qquad \alpha = e, \mu, \tau \ . \tag{1.1}$$

If the number of massive neutrinos is more than three, we also have

$$(\nu_{sR})^c = \sum_i U_{si} \nu_{iL} \quad , \qquad s = s_1, s_2, \dots \ , \tag{1.2}$$

where $(\nu_{sR})^c = \mathcal{C} \overline{\nu}_{sR}^T$ ($\mathcal{C}$ is the matrix of charge-conjugation), ν_{sR} are right-handed (sterile) fields, U is a unitary mixing matrix and the index s denotes the last (one or more, depending on the number of the sterile fields) rows of U. From Eqs.(1.1) and (1.2) it follows that not only transitions among active neutrino states are possible, but also transitions among active and sterile states. We will consider the case of two neutrino groups with close masses,

$$m_1 \leq \dots \leq m_M \qquad \text{and} \qquad m_{M+1} \leq \dots \leq m_N \ , \tag{1.3}$$

and we will assume that

$$m_j \ll m_k \ , \qquad \text{with} \qquad \begin{cases} j = 1, \dots, M \ , \\ k = M+1, \dots, N \ . \end{cases} \tag{1.4}$$

We will also assume that in the experiments with terrestrial neutrinos we have

$$\frac{\left(m_j^2 - m_{j'}^2\right) L}{2 \, p} \ll 1 \ , \qquad j, j' \leq M \ , \tag{1.5}$$

$$\frac{\left(m_k^2 - m_{k'}^2\right) L}{2 \, p} \ll 1 \ , \qquad k, k' > M \ , \tag{1.6}$$

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where L is the distance between the source and the detector and p is the neutrino momentum. Taking into account Eqs.(1.5) and (1.6), the amplitude of $\nu_\alpha \to \nu_\beta$ transitions (here ν_α and ν_β are the active or sterile neutrinos) for the short-baseline terrestrial experiments is given by

$$
\mathcal{A}_{\nu_\alpha \to \nu_\beta} = \mathrm{e}^{-iE_1 t} \sum_{j=1}^{N} U_{\beta j} \, U_{\alpha j}^* \, \mathrm{e}^{-i(E_j - E_1)t}
$$

$$
\simeq \mathrm{e}^{-iE_1 t} \left\{ \sum_{j=1}^{M} U_{\beta j} U_{\alpha j}^* + \exp\left(-i\frac{\Delta m^2 L}{2p}\right) \sum_{k=M+1}^{N} U_{\beta k} U_{\alpha k}^* \right\} , \tag{1.7}
$$

with $\Delta m^2 \equiv m_N^2 - m_1^2$, $E_i = \sqrt{p^2 + m_i^2} \simeq p + m_i^2/2p$ and $t \simeq L$. Furthermore, using the unitarity relation

$$
\sum_{j=1}^{M} U_{\beta j} U_{\alpha j}^* = \delta_{\alpha\beta} - \sum_{k=M+1}^{N} U_{\beta k} U_{\alpha k}^* , \tag{1.8}
$$

we can rewrite the transition amplitude $\mathcal{A}_{\nu_\alpha \to \nu_\beta}$ in the form

$$
\mathcal{A}_{\nu_\alpha \to \nu_\beta} \simeq \mathrm{e}^{-iE_1 T} \left\{ \delta_{\alpha\beta} + \left[\exp\left(-i\frac{\Delta m^2 L}{2p}\right) - 1\right] \sum_{k=M+1}^{N} U_{\beta k} U_{\alpha k}^* \right\} . \tag{1.9}
$$

From Eq.(1.9), for the probability of $\nu_\alpha \to \nu_\beta$ transitions with $\beta \neq \alpha$ we obtain the following expression:

$$
P_{\nu_\alpha \to \nu_\beta} = \frac{1}{2} A_{\nu_\alpha;\nu_\beta} \left(1 - \cos\frac{\Delta m^2 L}{2p}\right) , \tag{1.10}
$$

where

$$
A_{\nu_\alpha;\nu_\beta} = 4 \left| \sum_{k=M+1}^{N} U_{\beta k} U_{\alpha k}^* \right|^2 \tag{1.11}
$$

is the amplitude of $\nu_\alpha \to \nu_\beta$ oscillations.

The expression for the survival probability of ν_α can be obtained from Eq.(1.10) and the conservation of the total probability:

$$
P_{\nu_\alpha \to \nu_\alpha} = 1 - \sum_{\beta \neq \alpha} P_{\nu_\alpha \to \nu_\beta} = 1 - \frac{1}{2} B_{\nu_\alpha;\nu_\alpha} \left(1 - \cos\frac{\Delta m^2 L}{2p}\right) , \tag{1.12}
$$

where the oscillation amplitude $B_{\nu_\alpha;\nu_\alpha}$ is given by

$$
B_{\nu_\alpha;\nu_\alpha} = \sum_{\beta \neq \alpha} A_{\nu_\alpha;\nu_\beta} . \tag{1.13}
$$

Using the unitarity of the mixing matrix, from Eqs.(1.11) and (1.13) we obtain

$$B_{\nu_\alpha;\nu_\alpha} = 4 \left(\sum_{k=M+1}^{N} |U_{\alpha k}|^2 \right) \left(1 - \sum_{k=M+1}^{N} |U_{\alpha k}|^2 \right) . \tag{1.14}$$

Let us notice that from Eq.(1.11) we have

$$A_{\nu_\alpha;\nu_\beta} = A_{\nu_\beta;\nu_\alpha} . \tag{1.15}$$

From Eqs.(1.10) and (1.15) it follows that

$$P_{\nu_\alpha \to \nu_\beta} = P_{\nu_\beta \to \nu_\alpha} . \tag{1.16}$$

Due to CPT invariance we have (see, for example, Refs. [14,36,24])

$$P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} . \tag{1.17}$$

Thus, if only one squared-mass difference between the heaviest and lightest neutrinos is relevant for the oscillations of the terrestrial neutrinos, even in the case of non-conservation of CP in the lepton sector, the transition probabilities of neutrinos ($\nu_\alpha \to \nu_\beta$) and antineutrinos ($\bar{\nu}_\alpha \to \bar{\nu}_\beta$) are equal:

$$P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} . \tag{1.18}$$

Before closing this Section, let us remark that from the unitarity relation (1.8) it follows that the amplitude (1.11) of $\nu_\alpha \leftrightarrows \nu_\beta$ oscillations can also be written as

$$A_{\nu_\alpha;\nu_\beta} = 4 \left| \sum_{j=1}^{M} U_{\beta j} U_{\alpha j}^* \right|^2 . \tag{1.19}$$

In an analogous way, from Eqs.(1.8) it follows that the oscillation amplitudes $B_{\nu_\alpha;\nu_\alpha}$ given in Eq.(1.14) can also be written as

$$B_{\nu_\alpha;\nu_\alpha} = 4 \left(\sum_{j=1}^{M} |U_{\alpha j}|^2 \right) \left(1 - \sum_{j=1}^{M} |U_{\alpha j}|^2 \right) . \tag{1.20}$$

Thus, the expressions for the oscillation amplitudes $A_{\nu_\alpha;\nu_\beta}$ and $B_{\nu_\alpha;\nu_\alpha}$ can be written with the sum over the indexes of the neutrinos of the second group (Eqs.(1.11) and (1.14)) or with the sum over the indexes of the neutrinos of the first group (Eqs.(1.19) and (1.20)).

II. MIXING OF THREE MASSIVE NEUTRINOS

Let us consider first the schemes with mixing of three massive neutrino fields. The case of a neutrino mass hierarchy

$$m_1 \ll m_2 \ll m_3 . \tag{2.1}$$

was considered recently in Refs. [26,37–42]. In this case the oscillations of the terrestrial neutrinos are determined by three parameters: Δm^2 and the two mixing parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$. In Ref. [38,39] it was shown that, if a hierarchy of couplings is realized in the lepton sector (i.e. $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ are small), the LSND result is not compatible with the results of all the other reactor and accelerator experiments on the search for neutrino oscillations. Instead, the result of the LSND experiment is compatible with the results of all the other experiments in the scheme with the neutrino mass hierarchy (2.1) if $|U_{e3}|^2$ is small and $|U_{\mu 3}|^2$ is large (close to one). If massive neutrinos are Majorana particles, neutrinoless double-beta decay is allowed and for $\Delta m^2 \gtrsim 5\,\mathrm{eV}^2$ this process can have a rate in the region of sensitivity of the next generation of experiments [26,39].

Here we will consider another possible scheme with three massive neutrinos and one Δm^2 relevant for the oscillations of the terrestrial neutrinos. We will assume the following relation among the three neutrino masses:

$$m_1 \ll m_2 \simeq m_3 \tag{2.2}$$

This scheme was recently discussed in Refs. [26–28]. In favor of such a scheme there are some cosmological arguments [32] and some astrophysical arguments concerning the r-process production of heavy elements in the neutrino-heated ejecta of supernovae [35].

We will assume that the squared-mass difference $\Delta m^2_{32} \equiv m^2_3 - m^2_2$ is small and is relevant for the suppression of the flux of solar ν_e's on the earth. In this case, from the general formulas given in Section I it follows that the oscillations of the terrestrial neutrinos are characterized by the values of $\Delta m^2 \equiv m^2_3 - m^2_1$ and two mixing parameters $|U_{e1}|^2$ and $|U_{\mu 1}|^2$. The probabilities of $\nu_\alpha \to \nu_\beta$ $(\beta \neq \alpha)$ and $\nu_\alpha \to \nu_\alpha$ transitions are given by Eqs.(1.10) and (1.12) with the following oscillation amplitudes:

$$A_{\nu_\alpha;\nu_\beta} = 4|U_{\beta 1}|^2|U_{\alpha 1}|^2 \ , \tag{2.3}$$
$$B_{\nu_\alpha;\nu_\alpha} = 4|U_{\alpha 1}|^2 \left(1 - |U_{\alpha 1}|^2\right) \ . \tag{2.4}$$

Let us consider first the constraints in the scheme under consideration following from the results of reactor and accelerator disappearance experiments. We will use the exclusion plots obtained in the $\bar{\nu}_e \to \bar{\nu}_e$ Bugey reactor experiment [43] and in the $\nu_\mu \to \nu_\mu$ CDHS and CCFR84 accelerator experiments [44,45]. At fixed values of Δm^2, the allowed values of the oscillation amplitudes $B_{\nu_e;\nu_e}$ and $B_{\nu_\mu;\nu_\mu}$ are constrained by

$$B_{\nu_\alpha;\nu_\alpha} \leq B^0_{\nu_\alpha;\nu_\alpha} \qquad (\alpha = e, \mu) \ . \tag{2.5}$$

The values of $B^0_{\nu_e;\nu_e}$ and $B^0_{\nu_\mu;\nu_\mu}$ can be obtained from the corresponding exclusion curves. We will consider values of Δm^2 in the interval

$$0.5\,\mathrm{eV}^2 \lesssim \Delta m^2 \lesssim 10^2\,\mathrm{eV}^2 \ , \tag{2.6}$$

that covers the range where positive indications in favor of $\bar{\nu}_\mu \leftrightarrows \bar{\nu}_e$ oscillations were reported by the LSND collaboration [21]. In this range of Δm^2 the quantities $B^0_{\nu_e;\nu_e}$ and $B^0_{\nu_\mu;\nu_\mu}$ are smaller than 0.16 and 0.40, respectively. From Eq.(2.4) it follows that the parameters $|U_{\alpha 1}|^2$ at fixed values of Δm^2 must satisfy one of the following inequalities:

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$$|U_{\alpha 1}|^2 \le a_\alpha^0 \tag{2.7}$$

or

$$|U_{\alpha 1}|^2 \ge 1 - a_\alpha^0 \,, \tag{2.8}$$

with

$$a_\alpha^0 \equiv \frac{1}{2}\left(1 - \sqrt{1 - B^0_{\nu_\alpha;\nu_\alpha}}\right) \,. \tag{2.9}$$

In the interval of Δm^2 given in Eq.(2.6) the values of a_e^0 and a_μ^0 are smaller than 0.04 and 0.1, respectively. Thus, in the wide range of Δm^2 under consideration the mixing parameters $|U_{e1}|^2$ and $|U_{\mu 1}|^2$ can either be small or large (close to one).

We have assumed that Δm^2_{32} is relevant for the suppression of the flux of solar neutrinos on the earth and that $\Delta m^2_{32} \ll \Delta m^2$. In this case the survival probability of the solar ν_e's is given by

$$P_{\nu_e \to \nu_e} = \left(1 - |U_{e1}|^2\right)^2 P^{(2,3)}_{\nu_e \to \nu_e} + |U_{e1}|^4 \,, \tag{2.10}$$

where $P^{(2,3)}_{\nu_e \to \nu_e}$ is the survival probability due to the mixing of ν_e with ν_2 and ν_3 (Eq.(2.10) can be obtained with the method presented in Ref. [46]). From Eq.(2.10) and from the results of the Bugey disappearance experiment it follows that, in the case of a large $|U_{e1}|^2$, we have $P_{\nu_e \to \nu_e} \ge 0.92$ for all values of the neutrino energy. Such a large lower bound of the ν_e survival probability is not compatible with the results of solar neutrino experiments [1–4,10].

It is obvious that, due to the unitarity constraint $|U_{e1}|^2 + |U_{\mu 1}|^2 \le 1$, the mixing parameters $|U_{e1}|^2$ and $|U_{\mu 1}|^2$ cannot be both large. Thus, from the results of reactor and accelerator disappearance experiments and from the results of solar neutrino experiments it follows that the values of the parameters $|U_{e1}|^2$ and $|U_{\mu 1}|^2$ can lie in one of the following two regions:

   I. The region of small $|U_{e1}|^2$ and $|U_{\mu 1}|^2$.

   II. The region of small $|U_{e1}|^2$ and large $|U_{\mu 1}|^2$.

The fact that the parameter $|U_{e1}|^2$ is small can have important implications for ^{3}H β-decay experiments and for the experiments on the search for neutrinoless double-beta decay ($(\beta\beta)_{0\nu}$-decay).

The electron spectrum in the decay $^3\text{H} \to {}^3\text{He} + e^- + \bar\nu_e$ is given by

$$\frac{\mathrm{d}N}{\mathrm{d}E} = C\, p_e\, E_e\, (Q - T)\, F(E_e) \sum_i |U_{ei}|^2 \sqrt{(Q - T)^2 - m_i^2} \,. \tag{2.11}$$

Here Q is the energy release, p_e and E_e are the electron momentum and energy, $T = E_e - m_e$, $F(E_e)$ is the Fermi function and C is a constant. Taking into account that $|U_{e1}|^2 \ll 1$ and $m_2 \simeq m_3$, from Eq.(2.11) we have

$$\frac{\mathrm{d}N}{\mathrm{d}E} = C\, p_e\, E_e\, (Q - T)\, F(E_e) \sqrt{(Q - T)^2 - m_3^2} \,. \tag{2.12}$$

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This is the usual expression for the ^{3}H β-decay spectrum from which information about the value of the "electron neutrino mass" m_{ν_e} is extracted. The upper limit for m_{ν_e} obtained in the latest experiment [47] is $m_{\nu_e} \leq 4.5\,\text{eV}$, which implies that

$$\Delta m^2 \lesssim 20\,\text{eV}^2 \tag{2.13}$$

If ν_2 and ν_3 contribute to the dark matter in the universe, the masses $m_2 \simeq m_3$ are in the eV range. Thus, further improvements of the sensitivity of ^{3}H β-decay experiments could have important implications for the dark matter problem.

If massive neutrinos are Majorana particles, neutrinoless double-beta decay of some even-even nuclei is allowed. The matrix element of $(\beta\beta)_{0\nu}$ decay is proportional to [48]

$$\langle m \rangle = \sum_i U_{ei}^2 \, m_i \ . \tag{2.14}$$

From the results of the experiments on the search for $(\beta\beta)_{0\nu}$ decay it follows that $|\langle m \rangle| \lesssim 1 - 2\,\text{eV}$ (see the review in Ref. [49]). The expected sensitivity of experiments of the next generation is $|\langle m \rangle| \simeq$ a few $10^{-1}\,\text{eV}$ (see Ref. [50]).

In the model under consideration

$$\langle m \rangle \simeq \left(U_{e2}^2 + U_{e3}^2 \right) m_3 \ . \tag{2.15}$$

We will consider the general case of non-conservation of CP in the lepton sector. The case of CP conservation was already discussed in Ref. [26].

Let us write U_{ek} as

$$U_{ek} = |U_{ek}| \, \text{e}^{i\phi_k} \ . \tag{2.16}$$

If CP is conserved in the lepton sector we have [51]

$$U_{ek} = U_{ek}^* \, \eta_k \ , \tag{2.17}$$

where $\eta_k = \pm i$ is the CP parity of the Majorana neutrino with mass m_k. From Eqs.(2.16) and (2.17) it follows that in the case of CP invariance we have

$$\phi_k = \pm \frac{\pi}{4} \ . \tag{2.18}$$

Furthermore, taking into account that $|U_{e1}|^2 \ll 1$, from the unitarity of the mixing matrix we have

$$|U_{e2}|^2 + |U_{e3}|^2 \simeq 1 \ . \tag{2.19}$$

From Eqs.(2.15), (2.16) and (2.19) we obtain

$$|\langle m \rangle| \simeq m_3 \sqrt{1 - 4\,|U_{e2}|^2\,|U_{e3}|^2\,\sin^2 \Phi} \ , \tag{2.20}$$

with

$$\Phi \equiv |\phi_3 - \phi_2| \ . \tag{2.21}$$

If CP is conserved, we have $\Phi = 0$ or $\pi/2$ for the same or opposite CP parities of ν_2 and ν_3, respectively.

Due to Eq.(2.19) we can write

$$|U_{e2}| \simeq \cos\theta \qquad \text{and} \qquad |U_{e3}| \simeq \sin\theta \ . \tag{2.22}$$

With the help of Eqs.(2.20) and (2.22) we obtain

$$\frac{|\langle m \rangle|^2}{m_3^2} \simeq 1 - \sin^2 2\theta \, \sin^2 \Phi \ . \tag{2.23}$$

From Eq.(2.23) it follows that

$$1 - \sin^2 2\theta \le \frac{|\langle m \rangle|^2}{m_3^2} \le 1 \ . \tag{2.24}$$

The boundary values of $|\langle m \rangle|^2 / m_3^2$ correspond to CP conservation in the lepton sector: the upper (lower) bound corresponds to the case of equal (opposite) CP parities of ν_2 and ν_3.

Information about the value of the parameter $\sin^2 2\theta$ can be obtained from the results of solar neutrino experiments. If $\sin^2 2\theta \ll 1$, which corresponds to the MSW solution with a small mixing angle, we have

$$|\langle m \rangle| \simeq m_3 \ , \tag{2.25}$$

independently from the value of Φ and from the conservation of CP. In the case of a large value of the parameter $\sin^2 2\theta$, which correspond to the MSW solution with a large mixing angle or to the vacuum oscillation solution, in the future it will be possible to obtain information about the violation of CP in the lepton sector if both the ^{3}H β-decay and $(\beta\beta)_{0\nu}$ decay experiments will obtain positive results. In fact, from the measurement of m_3 in ^{3}H β-decay experiments, of $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ decay experiments, and of $\sin^2 2\theta$ in the solar neutrino experiments, with the help of Eq.(2.23) it will be possible to determine the value of $\sin^2 \Phi$ (if $\sin^2 2\theta$ is large). Let us emphasize that in the case of CP conservation the relative CP parities of ν_2 and ν_3 can be determined: the CP parities are equal if $\sin^2 \Phi = 0$ and opposite if $\sin^2 \Phi = 1$.

Now we will discuss the implications in the model under consideration of the results of the appearance neutrino oscillation experiments. We will use the exclusion plots, that were obtained in the BNL E776 experiment [52] searching for $\nu_\mu \to \nu_e$ transitions, in the FNAL E531 and CCFR95 experiments [53,54] searching for $\nu_\mu \to \nu_\tau$ transitions and in the FNAL E531 experiment [53] searching for $\nu_e \to \nu_\tau$ transitions. We can use the results obtained in Ref. [38,39] for the case of mixing of three neutrinos and the mass hierarchy (2.1) if we make the change

$$|U_{\alpha 3}|^2 \to |U_{\alpha 1}|^2 \ . \tag{2.26}$$

In the following two subsections we will briefly present the main results for the two allowed regions of the values of the parameters $|U_{e1}|^2$ and $|U_{\mu 1}|^2$.

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A. The region I of small $|U_{e1}|^2$ and $|U_{\mu 1}|^2$

From Eq.(2.3) it follows that the oscillation amplitude $A_{\nu_\mu;\nu_e}$ is quadratic in the small quantities $|U_{e1}|^2$ and $|U_{\mu 1}|^2$, whereas the oscillation amplitudes $A_{\nu_\mu;\nu_\tau}$ and $A_{\nu_e;\nu_\tau}$ depend only linearly on these quantities. Thus, we expect that in the region I $\nu_\mu \leftrightarrows \nu_e$ oscillations are suppressed.

From Eq.(2.3), at fixed values of Δm^2 in the interval under consideration, we have

$$A_{\nu_\mu;\nu_e} \leq 4\, a_e^0\, a_\mu^0 \, , \tag{2.27}$$

with a_e^0 and a_μ^0 given by Eq.(2.9). In Fig.1 we have plotted the curve that represents this upper bound obtained from the results of the Bugey, CDHS and CCFR84 experiments (the curve passing through the circles). In Fig.1 we have also plotted the exclusion curves obtained in the BNL E776 [52] (dash-dotted line) and KARMEN [55] (dash-dot-dotted line) experiments on the search of $\nu_\mu \to \nu_e$ transitions. The region allowed by the results of the LSND experiment is shown in Fig.1 as the shadowed region between the two solid lines. Taking into account that $A_{\nu_\mu;\nu_e} \leq B_{\nu_e;\nu_e}$ (see Eq.(1.13)), we also plotted in Fig.1 the exclusion curve for $B_{\nu_e;\nu_e}$ found in the Bugey experiment (dashed line). It can be seen from the figure that, for small values of Δm^2 ($\Delta m^2 \lesssim 0.5\,\mathrm{eV}^2$), this bound on $A_{\nu_\mu;\nu_e}$ is stronger than the direct bound obtained by the BNL E776 and KARMEN experiments. It is also clear from the figure that this bound is not compatible with the result of the LSND experiment for $\Delta m^2 \lesssim 0.2\,\mathrm{eV}^2$.

It can be seen from Fig.1 that, in the range of Δm^2 under consideration, with the exception of the region $10\,\mathrm{eV}^2 \lesssim \Delta m^2 \lesssim 60\,\mathrm{eV}^2$, the limits on the oscillation amplitude $A_{\nu_\mu;\nu_e}$ that can be obtained from the results of disappearance experiments are more stringent than the limits obtained in the direct experiments searching for $\nu_\mu \to \nu_e$ transitions.

For $\Delta m^2 \gtrsim 4\,\mathrm{eV}^2$ we can obtain even stronger limits on the oscillation amplitude $A_{\nu_\mu;\nu_e}$ if we take into account the results of the FNAL E531 and CCFR95 experiments on the search for $\nu_\mu \to \nu_\tau$ transitions. In fact, in the linear approximation over the small quantities $|U_{e1}|^2$ and $|U_{\mu 1}|^2$, we have

$$|U_{\mu 1}|^2 \simeq \frac{A_{\nu_\mu;\nu_\tau}}{4} \, . \tag{2.28}$$

From Eqs.(2.3) and (2.28) we obtain the following upper bound:

$$A_{\nu_\mu;\nu_e} \lesssim a_e^0\, A_{\nu_\mu;\nu_\tau}^0 \, , \tag{2.29}$$

where $A_{\nu_\mu;\nu_\tau}^0$ is the upper bound for the amplitude of $\nu_\mu \leftrightarrows \nu_\tau$ oscillations given by the exclusion curves obtained in the FNAL E531 and CCFR95 experiments. The bound (2.29) on the oscillation amplitude $A_{\nu_\mu;\nu_e}$ obtained from the results of the Bugey, FNAL E531 and CCFR95 experiments is presented in Fig.1 (the curve passing through the triangles).

From Fig.1 it can be seen that the results of reactor and accelerator disappearance experiments, together with the results of $\nu_\mu \to \nu_\tau$ appearance experiments allow us to exclude practically all the region of the parameters Δm^2 and $A_{\nu_\mu;\nu_e}$ that is allowed by the LSND experiment. Let us stress that this result is valid if the parameters $|U_{e1}|^2$ and $|U_{\mu 1}|^2$ are both small.

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In Fig.1, we have also shown the region in the $A_{\nu_\mu;\nu_e}$–Δm^2 plane that will be explored when the projected sensitivity of the CHORUS [56] and NOMAD [57] experiments, which are searching for $\nu_\mu \to \nu_\tau$ transitions, is reached (the region limited by the line passing through the squares).

B. The region II of small $|U_{e1}|^2$ and large $|U_{\mu1}|^2$

From the discussion in Section II A it follows that, if the indications in favor of $\nu_\mu \leftrightarrows \nu_e$ oscillations reported by the LSND collaboration [21] are confirmed by future experiments, the parameters $|U_{e1}|^2$ and $|U_{\mu1}|^2$ must satisfy the inequalities

$$|U_{e1}|^2 \leq a_e^0 \qquad \text{and} \qquad |U_{\mu1}|^2 \geq 1 - a_\mu^0 . \tag{2.30}$$

with the quantities a_e^0 and a_μ^0 given by Eq.(2.9).

Taking into account the unitarity of the mixing matrix and Eq.(2.30) we have

$$|U_{e1}|^2 \leq 1 - |U_{\mu1}|^2 \leq a_\mu^0 . \tag{2.31}$$

From Eqs.(2.30) and (2.31) we have

$$|U_{e1}|^2 \leq \text{Min} \left[a_e^0, a_\mu^0 \right] . \tag{2.32}$$

The inequalities (2.30) and the unitarity of the mixing matrix imply that also $|U_{\tau1}|^2$ is small:

$$|U_{\tau1}|^2 \leq 1 - |U_{\mu1}|^2 \leq a_\mu^0 . \tag{2.33}$$

Taking into account the inequalities (2.30)–(2.33), from Eq.(2.3) it follows that the oscillation amplitude $A_{\nu_e;\nu_\tau}$ is quadratic in the small quantities $|U_{e1}|^2$ and $|U_{\tau1}|^2$, whereas the oscillation amplitudes $A_{\nu_\mu;\nu_e}$ and $A_{\nu_\mu;\nu_\tau}$ are linear in the same small quantities. For the oscillation amplitude $A_{\nu_e;\nu_\tau}$ we have the following upper bound:

$$A_{\nu_e;\nu_\tau} \leq 4 \, \text{Min} \left[a_e^0, a_\mu^0 \right] a_\mu^0 . \tag{2.34}$$

In Fig.2 we have plotted the upper bound for the oscillation amplitude $A_{\nu_e;\nu_\tau}$ obtained from Eq.(2.34) using the results of the Bugey, CDHS and CCFR84 experiments (the curve passing through the triangles). The solid line in Fig.2 is the exclusion curve obtained in the FNAL E531 experiment. Taking into account the unitarity constraint $A_{\nu_e;\nu_\tau} \leq B_{\nu_e;\nu_e}$, in Fig.2 we have also plotted the exclusion curve obtained in the Bugey experiment. From Fig.2 it can be seen that in all the considered range of Δm^2 the upper bound for $A_{\nu_e;\nu_\tau}$ given by Eq.(2.34) is very small, varying from about 10^{-3} to about 10^{-2} for $\Delta m^2 \gtrsim 0.3 \, \text{eV}^2$.

Additional limits on the oscillation amplitude $A_{\nu_e;\nu_\tau}$ can be obtained by taking into account the results of the experiments on the search for $\nu_\mu \leftrightarrows \nu_e$ and $\nu_\mu \leftrightarrows \nu_\tau$ oscillations. In the linear approximation in the small quantities $|U_{e1}|^2$, $(1 - |U_{\mu1}|^2)$ and $|U_{\tau1}|^2$, from Eq.(2.3) we have

$$|U_{e1}|^2 \simeq \frac{A_{\nu_\mu;\nu_e}}{4} , \tag{2.35}$$

$$|U_{\tau1}|^2 \simeq \frac{A_{\nu_\mu;\nu_\tau}}{4} . \tag{2.36}$$

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With the help of Eqs.(2.33) and (2.35) we obtain the following upper bound for the oscillation amplitude $A_{\nu_e;\nu_\tau}$:

$$A_{\nu_e;\nu_\tau} \leq A^0_{\nu_\mu;\nu_e} \, a^0_\mu \, . \tag{2.37}$$

In Fig.2 we have plotted the corresponding boundary curve obtained from the results of the CDHS, CCFR84 and BNL E776 experiments (the curve passing through the squares). From this figure it can be seen that for $\Delta m^2 \gtrsim 1\,\mathrm{eV}^2$ the upper bound on the oscillation amplitude $A_{\nu_e;\nu_\tau}$ varies from 2×10^{-5} to 5×10^{-4} and is more stringent than that obtained from the results of disappearance experiments using Eq.(2.34).

Finally, from Eqs.(2.35) and (2.36) we have

$$A_{\nu_e;\nu_\tau} \lesssim \frac{A^0_{\nu_\mu;\nu_e} \, A^0_{\nu_\mu;\nu_\tau}}{4} \tag{2.38}$$

The corresponding boundary curve obtained from the results of the BNL E776, FNAL E531 and CCFR95 experiments is presented in Fig.2 (the curve passing through the circles). From this figure it can be seen that for $\Delta m^2 \gtrsim 4\,\mathrm{eV}^2$ the amplitude of $\nu_e \leftrightarrows \nu_\tau$ oscillations is smaller than 3×10^{-5}.

Thus, if the parameter $|U_{e1}|^2$ is small and the parameter $|U_{\mu1}|^2$ is large, $\nu_e \leftrightarrows \nu_\tau$ oscillations are strongly suppressed. On the other hand, there are no constraints on the amplitudes of $\nu_\mu \leftrightarrows \nu_e$ and $\nu_\mu \leftrightarrows \nu_\tau$ oscillations. As is well known, new experiments on the search for $\nu_\mu \to \nu_\tau$ transitions are under way at CERN (CHORUS [56] and NOMAD [57]). Experiments on the search for $\nu_\mu \to \nu_e$ transitions are continuing at Los Alamos (LSND [21]) and at Rutherford Appleton Laboratory (KARMEN [55]).

Let us conclude this Section with the following remarks. In the general case of mixing, the flavor neutrinos ν_e, ν_μ and ν_τ are not particles with a definite mass. Effective masses of the flavor neutrinos can be introduced in the framework of specific models. This is the case of the model under consideration. If the mixing parameters are in the region I ν_e and ν_μ are heavy and ν_τ is the lightest neutrino: $m_{\nu_e} \simeq m_3$, $m_{\nu_\mu} \simeq m_3$ and $m_{\nu_\tau} \simeq m_1$. If the mixing parameters are in the region II ν_e and ν_τ are heavy and ν_μ is the lightest neutrino: $m_{\nu_e} \simeq m_3$, $m_{\nu_\tau} \simeq m_3$ and $m_{\nu_\mu} \simeq m_1$. Such inverted neutrino mass spectra are compatible with the r-process production of heavy elements in the neutrino-heated ejecta of supernovae [35].

III. MIXING OF FOUR MASSIVE NEUTRINOS

All the existing indications in favor of neutrino masses and mixing (solar neutrinos, atmospheric neutrinos, dark matter, LSND) require at least three different scales of squared-mass differences. In this section we will consider short-baseline oscillations of the terrestrial neutrinos in a scheme with mixing of four massive neutrinos. We will consider a specific scheme [29–34] with two light neutrinos ν_1, ν_2 and two neutrinos ν_3, ν_4 with masses in the eV range. We will assume that the value of $\Delta m^2_{43} \equiv m^2_4 - m^2_3$ is relevant for the suppression of the flux of solar ν_e's and the value of $\Delta m^2_{21} \equiv m^2_2 - m^2_1$ could be relevant for the explanation of the atmospheric neutrino anomaly. Thus, we have

$$\Delta m_{43}^2 \ll \Delta m_{21}^2 \ll \Delta m^2 \ , \tag{3.1}$$

with $\Delta m^2 \equiv m_4^2 - m_1^2$.

The important difference between the model considered here and that considered in Section II is that in this model transitions of active neutrinos into sterile states are possible. The transition probabilities between different states are given by the general expressions (1.10) and (1.12) with the following oscillation amplitudes:

$$A_{\nu_\alpha;\nu_\beta} = 4 \left| \sum_{j=1,2} U_{\beta j} U_{\alpha j}^* \right|^2 \ , \tag{3.2}$$

$$B_{\nu_\alpha;\nu_\alpha} = 4\, b_\alpha \left(1 - b_\alpha \right) \ , \tag{3.3}$$

where

$$b_\alpha \equiv \sum_{j=1,2} |U_{\alpha j}|^2 \tag{3.4}$$

As in Section II, we start with the implications in the scheme under consideration of the results of the reactor and accelerator disappearance experiments. It is clear that we can use the results presented in Section II. We have

$$b_\alpha \leq a_\alpha^0 \tag{3.5}$$

or

$$b_\alpha \geq 1 - a_\alpha^0 \ , \tag{3.6}$$

with $\alpha = e, \mu$. The quantities a_α^0 are determined by Eq.(2.9). For each value of Δm^2 we determined the value of a_e^0 and a_μ^0 from the exclusion plots obtained in the Bugey [43], CDHS [44] and CCFR84 [45] experiments. We will consider values of Δm^2 in the interval $0.5\,\mathrm{eV}^2 \lesssim \Delta m^2 \lesssim 10^2\,\mathrm{eV}^2$, where the quantities a_e^0 and a_μ^0 are smaller than 0.1.

Thus, the values of the parameters b_e and b_μ can lie in one of the following four regions:

I. The region of small b_e and b_μ.

II. The region of small b_e and large b_μ.

III. The region of large b_e and small b_μ.

IV. The region of large b_e and b_μ.

If the neutrino masses satisfy the relation (3.1) and Δm_{43}^2 is relevant for the suppression of the solar ν_e flux, the probability of solar ν_e's to survive is given by (see the Appendix A)

$$P_{\nu_e \to \nu_e} = \left(1 - \sum_{j=1,2} |U_{ej}|^2 \right)^2 P_{\nu_e \to \nu_e}^{(3,4)} + \sum_{j=1,2} |U_{ej}|^4 \ , \tag{3.7}$$

where $P_{\nu_e \to \nu_e}^{(3,4)}$ is the survival probability due to the mixing of ν_e with ν_3 and ν_4. If the parameter b_e is large, for the survival probability we have

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$$P_{\nu_e \to \nu_e} \simeq \sum_{j=1,2} |U_{ej}|^4 \ . \tag{3.8}$$

Thus, if the parameters b_e and b_μ lie in the region III or in the region IV, the probability $P_{\nu_e \to \nu_e}$ practically does not depend on the neutrino energy. Moreover, taking into account that in the regions III and IV we have $\sum_{j=1,2} |U_{ej}|^2 \simeq 1$, from Eq.(3.8) we obtain

$$P_{\nu_e \to \nu_e} \gtrsim \frac{1}{2} \ . \tag{3.9}$$

The case of a constant $P_{\nu_e \to \nu_e}$ is disfavored by the solar neutrino data. In Ref. [17] it was shown that the solar neutrino data cannot be explained with a constant $P_{\nu_e \to \nu_e}$ if the fluxes of ^{8}B and ^{7}Be neutrinos are allowed to vary in rather wide intervals around the standard model values.

In both regions I and II we have $\sum_{k=3,4} |U_{ek}|^2 \simeq 1$. Thus, information about the value of the masses $m_3 \simeq m_4$ can be obtained from the investigation of the end-point part of the β-spectrum of ^{3}H decay (see Eq.(2.12)). If massive neutrinos are Majorana particles, neutrinoless double-beta decay is possible. It is clear that all the relations referring to this process derived in Section II in the framework of the scheme with three massive neutrinos (see Eqs.(2.14)–(2.24)) are also valid in the scheme under consideration.

In the following we will discuss the implications of the results of neutrino oscillations appearance experiments if the values of the parameters b_e and b_μ lie in the region I or in the region II.

A. The region I of small b_e and b_μ

In this region $\nu_\mu \leftrightarrows \nu_e$ oscillations are suppressed. In fact, using the Cauchy-Schwarz inequality, from Eqs.(3.2) and (3.4) we obtain

$$A_{\nu_\mu;\nu_e} \leq 4\,b_\mu\,b_e \ . \tag{3.10}$$

Taking into account that in this region $b_\mu \leq a_\mu^0$ and $b_e \leq a_e^0$, we obtain

$$A_{\nu_\mu;\nu_e} \leq 4\,a_\mu^0\,a_e^0 \ . \tag{3.11}$$

In Fig.3 we have plotted the curve that represent this upper bound obtained from the results of the Bugey, CDHS and CCFR84 experiments (the curve passing through the circles).

Let us consider now the neutrino oscillation appearance experiments. From the unitarity relation (1.13) we have

$$A_{\nu_\mu;\nu_\tau} \leq B_{\nu_\mu;\nu_\mu} \ , \tag{3.12}$$

$$A_{\nu_e;\nu_\tau} \leq B_{\nu_e;\nu_e} \ . \tag{3.13}$$

In the case of three massive neutrinos considered in Section II instead of these inequalities we had two approximate equalities, that allowed us to obtain additional strong limitations

for the amplitude of $\nu_\mu \leftrightarrows \nu_e$ oscillations (see Eq.(2.29)). In the model under consideration transitions of ν_μ and ν_e into sterile states are possible and, instead of approximate equalities such as Eq.(2.28), we have the inequalities

$$b_\mu \gtrsim \frac{A_{\nu_\mu;\nu_\tau}}{4} \ , \tag{3.14}$$

$$b_e \gtrsim \frac{A_{\nu_e;\nu_\tau}}{4} \ . \tag{3.15}$$

The inequality (3.14) can be useful if a positive result is found in the experiments on the search for $\nu_\mu \leftrightarrows \nu_\tau$ oscillations (CHORUS [56], NOMAD [57], COSMOS [58]).

An important difference between the model considered here and the one considered in Section II is that, in the case of mixing of four massive neutrinos, in the region I of small b_e and b_μ the result of the LSND experiment is compatible with the results of all the other neutrino oscillation experiments for $5\,\mathrm{eV}^2 \lesssim \Delta m^2 \lesssim 70\,\mathrm{eV}^2$.

B. The region II of small b_e and large b_μ

We will consider now the region II of small b_e and large b_μ. From Eq.(3.3) in the linear approximation in the small quantities b_e and $1 - b_\mu$ we have

$$b_e \simeq \frac{B_{\nu_e;\nu_e}}{4} \ , \tag{3.16}$$

$$1 - b_\mu \simeq \frac{B_{\nu_\mu;\nu_\mu}}{4} \ . \tag{3.17}$$

The essential difference of the model under consideration from the model with mixing of three neutrinos considered in the previous section is that, in the case of mixing of four massive neutrinos, $\nu_e \leftrightarrows \nu_\tau$ oscillations could be not suppressed in the region II. This is due to the fact that in the model under consideration transitions of active neutrinos into sterile states are possible. In fact, using the Cauchy-Schwarz inequality, from Eq.(3.2) we have

$$A_{\nu_e;\nu_\tau} \leq 4\,b_e\,b_\tau \ . \tag{3.18}$$

Furthermore, from the unitarity of the mixing matrix it follows that

$$\sum_\alpha b_\alpha = 2 \ . \tag{3.19}$$

From this equation we cannot conclude that in the region of small b_e and $1-b_\mu$ the parameter b_τ is small. Thus, the right-hand side of Eq.(3.18) could be not quadratic in small quantities (as it is in the case of mixing of three massive neutrinos) and $A_{\nu_e;\nu_\tau}$ could be not suppressed.

Let us notice that from the unitarity relation (1.13) and from Eqs.(3.16) and (3.17) we have the following lower bounds for b_e and $1 - b_\mu$

$$b_e \gtrsim \frac{A_{\nu_\mu;\nu_e}}{4} \ , \tag{3.20}$$

$$1 - b_\mu \gtrsim \frac{A_{\nu_\mu;\nu_e} + A_{\nu_\mu;\nu_\tau}}{4} \ . \tag{3.21}$$

C. Atmospheric neutrinos

Up to now we have considered the constraints on the value of the mixing parameters b_e and b_μ due to the results of the solar neutrino experiments and of the reactor and accelerator neutrino oscillation experiments. If the squared-mass difference Δm_{21}^2 is relevant for the oscillation of atmospheric neutrinos (which is natural in this scheme), we can obtain an additional constraint on b_e and b_μ from the results of the experiments on the detection of atmospheric neutrinos.

In the model under consideration the survival probability of atmospheric ν_α is given by (see the Appendix B)

$$
P_{\nu_\alpha \to \nu_\alpha} = \left(1 - \sum_{k=3,4} |U_{\alpha k}|^2 \right)^2 P_{\nu_\alpha \to \nu_\alpha}^{(1,2)} + \left(\sum_{k=3,4} |U_{\alpha k}|^2 \right)^2 , \tag{3.22}
$$

where $P_{\nu_\alpha \to \nu_\alpha}^{(1,2)}$ is the survival probability due to the mixing of ν_α with ν_1 and ν_2. In the region I, where both b_e and b_μ are small, we have

$$
P_{\nu_\mu \to \nu_\mu} = b_\mu^2 \, P_{\nu_\mu \to \nu_\mu}^{(1,2)} + (1 - b_\mu)^2 \simeq 1 , \tag{3.23}
$$

$$
P_{\nu_e \to \nu_e} = b_e^2 \, P_{\nu_e \to \nu_e}^{(1,2)} + (1 - b_e)^2 \simeq 1 . \tag{3.24}
$$

Thus, if the parameters b_e and b_μ are in the region I, the ratio of the observed neutrino-induced e-like and μ-like events in the atmospheric neutrino detectors should be equal to the expected ratio. This is in contradiction with the results of the Kamiokande [18], IMB [19] and Soudan [20] experiments. On the other hand, if the parameters b_e and b_μ are in the region II, from Eq.(3.22) we obtain

$$
P_{\nu_\mu \to \nu_\mu} \simeq P_{\nu_\mu \to \nu_\mu}^{(1,2)} , \tag{3.25}
$$

$$
P_{\nu_e \to \nu_e} \simeq 1 , \tag{3.26}
$$

and the results of the Kamiokande, IMB and Soudan experiments can be explained with $\nu_\mu \leftrightarrows \nu_\tau$ oscillations.

Therefore, if the atmospheric neutrino anomaly found in the Kamiokande, IMB and Soudan experiments is confirmed by future experiments, the values of the mixing parameters b_e and b_μ can lie only in the region II.

IV. CONCLUSIONS

We have considered short-baseline neutrino oscillations, ^{3}H β-decay and $(\beta\beta)_{0\nu}$-decay in two specific schemes with mixing of three and four massive neutrino fields, respectively.

Having in mind the experimental indications in favor of neutrino mass and mixing, we have assumed that in the case of three neutrinos the neutrino mass spectrum is composed of one very light neutrino with mass $m_1 \ll 1\,\text{eV}$ and two neutrinos with masses $m_2 \simeq m_3$ in the eV range and a squared-mass difference $m_3^2 - m_2^2$ which is relevant for the suppression of the flux of solar ν_e's. In this scheme, the oscillations of the terrestrial neutrinos are

determined by three parameters, $\Delta m^2 \equiv m_3^2 - m_1^2$ and the two mixing parameters $|U_{e1}|^2$ and $|U_{\mu 1}|^2$. From the analysis of the results of reactor and accelerator disappearance experiments and from the results of the solar neutrino experiments, it follows that only two regions of the values of the mixing parameters are allowed: I. $|U_{e1}|^2$ and $|U_{\mu 1}|^2$ are both small; II. $|U_{e1}|^2$ is small and $|U_{\mu 1}|^2$ is large (close to one). We have shown that in the region I the oscillations $\nu_\mu \leftrightarrows \nu_e$ are suppressed and the result of the LSND experiment is not compatible with the results of all the other neutrino oscillation experiments. Instead, in the region II the oscillations $\nu_e \leftrightarrows \nu_\tau$ are suppressed and we have obtained rather strong limits for the oscillation amplitude $A_{\nu_e;\nu_\tau}$ from the results of neutrino oscillation experiments. If the masses m_2 and m_3 are in the eV region, the effect of neutrino masses could be observed in the next generation of experiments on the measurement of the end-point part of the ^{3}H β-decay spectrum and in the next generation of experiments on the search for $(\beta\beta)_{0\nu}$-decay. We have shown that, if these experiment find a positive effect, a comparison of the results of the $(\beta\beta)_{0\nu}$-decay experiments and of the ^{3}H β-decay experiments could allow to obtain direct information about the CP violation in the lepton sector.

In the case of mixing of four massive neutrino fields we have assumed that the spectrum of neutrino masses is composed of two very light masses $m_1 \simeq m_2 \ll 1\,\mathrm{eV}$ with a value of $m_2^2 - m_1^2$ which could be relevant for the explanation of the atmospheric neutrino anomaly and two masses $m_3 \simeq m_4$ in the eV range with a value of $m_4^2 - m_3^2$ which is relevant for the suppression of the flux of solar ν_e's. Since in this model transitions from active to sterile states are possible, the unitarity of the mixing matrix does not put as strong constraints on the oscillation channels as in the case of three neutrinos. The survival probabilities of ν_e and ν_μ are determined by the two parameters $b_\alpha = \sum_{j=1,2} |U_{\alpha j}|^2$ (with $\alpha = e, \mu$). From the results of neutrino oscillation disappearance experiments and the results of solar neutrino experiments it follows that only two regions of values of the parameters b_e and b_μ are allowed: I. The region of small b_e and b_μ; II. The region of small b_e and large b_μ. Oscillations $\nu_\mu \leftrightarrows \nu_e$ are suppressed in the region I, but the LSND result is compatible with the results of other neutrino oscillation experiments if $\Delta m^2 = m_4^2 - m_1^2$ is in the range $5\,\mathrm{eV}^2 \lesssim \Delta m^2 \lesssim 70\,\mathrm{eV}^2$. If the parameters b_e and b_μ are in the region II, $\nu_e \leftrightarrows \nu_\tau$ oscillations could be not suppressed (unlike the case of three neutrinos). If the atmospheric neutrino anomaly is confirmed by future experiments, the region I will be excluded.

Both schemes considered here are in agreement with the cold and hot dark matter scenario with two practically degenerate neutrinos with masses in the eV range [32].

From the results of neutrino oscillation disappearance experiments and solar neutrino experiments it follows that in both schemes considered here the electron neutrino is "heavy". This means that both schemes are compatible with the constraints coming from the r-process nucleosynthesis in the neutrino-heated ejecta of supernovae [35].

In conclusion, let us remark that, if one of the neutrino mixing schemes considered here is realized in nature, i.e. there is no hierarchy of neutrino masses, the neutrino mass spectrum has a completely different character from the mass spectra of quarks and charged leptons. From our analysis of the results of neutrino oscillation experiments it follows that there is no hierarchy of couplings among generations in the lepton sector, i.e. the mixing matrix of neutrinos is completely different from that of quarks.

ACKNOWLEDGMENTS

APPENDIX A: SURVIVAL PROBABILITY OF SOLAR NEUTRINOS

In this appendix we will derive the formula (3.7) for the survival probability $P_{\nu_e \to \nu_e}$ of solar neutrinos in the case of mixing of four massive neutrino fields considered in Section III.

In general, the survival probability of solar ν_e's can be written as

$$P_{\nu_e \to \nu_e} = \left| \sum_{jk} U_{ej}^{(M)} \mathcal{A}_{jk} U_{ek}^* \right|^2 , \tag{A1}$$

where $U^{(M)}$ is the mixing matrix in matter in the point of ν_e production in the Sun and $\mathcal{A}_{jk}$ is the amplitude of $\nu_j^{\mathrm{m}} \to \nu_k$ transitions when the solar neutrinos travel from the central part of the Sun to the Earth, ν_j^{m} being the effective mass eigenstate neutrinos in matter in the point of ν_e production in the Sun (see, e.g., [36,24]). Equation (A1) is valid both for vacuum oscillations and for resonant MSW transitions of solar ν_e's. In the case of vacuum oscillations $U^{(M)} = U$, $\nu_j^{\mathrm{m}} = \nu_j$ and

$$\mathcal{A}_{jk} = \delta_{jk} \exp\left[-i \int_{x_i}^{x_f} E_j \, \mathrm{d}x \right] , \tag{A2}$$

where $E_j \simeq p + m_j^2 / 2p$, and x_i and x_f are the initial and final points of the neutrino trajectory.

In the model under consideration it is assumed that Δm_{43}^2 is relevant for the suppression of the flux of solar ν_e's and $\Delta m_{41}^2 \gg \Delta m_{21}^2 \gg \Delta m_{43}^2$. This implies that, if Δm_{43}^2 is in the range of the MSW solution of the solar neutrino problem, the transitions between $\nu_{4(3)}^{\mathrm{m}}$ and $\nu_{3(4)}$ in the Sun can be strongly affected by solar matter effects. At the same time, the resonance densities associated with $\Delta m_{41}^2 \simeq \Delta m_{42}^2 \simeq \Delta m_{31}^2 \simeq \Delta m_{32}^2$ and with Δm_{21}^2 are much bigger than the density in the Sun and therefore the evolution of the ν_1 and ν_2 states in the Sun is adiabatic (see the discussions in Refs. [59,60]). As a consequence we have

$$U_{ej}^{(M)} = U_{ej} \qquad \text{for} \qquad j = 1, 2 , \tag{A3}$$

and Eq.(A2) is valid for $j, k = 1, 2$ and for $j = 1, 2$ and $k = 3, 4$. Thus, the survival probability averaged over the neutrino energy spectrum and the region of neutrino production in the Sun can be written as

$$P_{\nu_e \to \nu_e} = \sum_{j=1,2} |U_{ej}|^4 + \left| \sum_{k,k'=3,4} U_{ek}^{(M)} \mathcal{A}_{kk'} U_{ek'}^* \right|^2 . \tag{A4}$$

Using the unitarity of the mixing matrix, we can write Eq.(A4) as

$$P_{\nu_e \to \nu_e} = \sum_{j=1,2} |U_{ej}|^4 + \left(1 - \sum_{j=1,2} |U_{ej}|^2\right)^2 P_{\nu_e \to \nu_e}^{(3,4)} , \tag{A5}$$

where

$$P_{\nu_e \to \nu_e}^{(3,4)} = \left| \sum_{k,k'=3,4} \frac{U_{ek}^{(M)}}{\sqrt{\sum_{k''=3,4} \left|U_{ek''}^{(M)}\right|^2}} \, \mathcal{A}_{kk'} \, \frac{U_{ek'}^*}{\sqrt{\sum_{k''=3,4} |U_{ek''}|^2}} \right|^2 . \tag{A6}$$

In the case of vacuum oscillations Eq.(A6) can be written as

$$P_{\nu_\alpha \to \nu_\alpha}^{(3,4)} = 1 - \frac{1}{2} \sin^2 2\theta \left(1 - \cos \frac{\Delta m_{43}^2 \, L}{2 \, p}\right) , \tag{A7}$$

which is the standard two-generation formula. Here the mixing angle θ is defined by

$$\cos \theta = \frac{|U_{e3}|}{\sqrt{\left(\sum_{k=3,4} |U_{ek}|^2\right)^2}} , \qquad \sin \theta = \frac{|U_{e4}|}{\sqrt{\left(\sum_{k=3,4} |U_{ek}|^2\right)^2}} . \tag{A8}$$

In the case of MSW resonant transitions Eq.(A6) can be written as

$$P_{\nu_e \to \nu_e}^{(3,4)} = \frac{1}{2} + \left(\frac{1}{2} - P_{34}\right) \cos 2\theta_M \cos 2\theta , \tag{A9}$$

which has the form of the standard two-generation formula. Here $P_{34} = |\mathcal{A}_{34}|^2$ is the probability of $\nu_3^m \to \nu_4$ transitions, which is equal to the probability of $\nu_4^m \to \nu_3$ transitions. The mixing angle θ in vacuum is defined by Eq.(A8) and the mixing angle θ_M in matter is defined by

$$\cos \theta_M = \frac{\left|U_{e3}^{(M)}\right|}{\sqrt{\left(\sum_{k=3,4} \left|U_{ek}^{(M)}\right|^2\right)^2}} , \qquad \sin \theta_M = \frac{\left|U_{e4}^{(M)}\right|}{\sqrt{\left(\sum_{k=3,4} \left|U_{ek}^{(M)}\right|^2\right)^2}} . \tag{A10}$$

APPENDIX B: SURVIVAL PROBABILITY OF ATMOSPHERIC NEUTRINOS

In this appendix we will derive the formula (3.22) for the survival probability $P_{\nu_\alpha \to \nu_\alpha}$ of atmospheric neutrinos in the case of mixing of four massive neutrino fields considered in Section III.

In general, the vacuum oscillation survival probability of ν_α is given by (see, for example, Refs. [14,36,24])

$$P_{\nu_\alpha \to \nu_\alpha} = \sum_{j,k} |U_{\alpha j}|^2 |U_{\alpha k}|^2 \exp\left(-i\frac{\Delta m_{jk}^2 L}{2p}\right) , \tag{B1}$$

with $\Delta m_{jk}^2 \equiv m_j^2 - m_k^2$.

In the model under consideration, for the atmospheric neutrinos we have

$$\exp\left(-i\frac{\Delta m_{43}^2 L}{2p}\right) \simeq 1 . \tag{B2}$$

After averaging over the neutrino energy and the distance between the points of neutrino production and detection, we have

$$\left\langle \exp\left(-i\frac{\Delta m_{jk}^2 L}{2p}\right) \right\rangle \simeq 0 \qquad \text{for} \qquad j = 1,2 \quad \text{and} \quad k = 3,4 . \tag{B3}$$

From Eqs.(B1)–(B3), for the survival probability of nu_α we obtain

$$P_{\nu_\alpha \to \nu_\alpha} = \left(\sum_{k=3,4} |U_{\alpha k}|^2\right)^2 + \sum_{j=1,2} |U_{\alpha j}|^4 + 2 |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \cos\frac{\Delta m_{21}^2 L}{2p} \tag{B4}$$

Using the unitarity relation (1.8) we have

$$\sum_{j=1,2} |U_{\alpha j}|^4 = \left(1 - \sum_{k=3,4} |U_{\alpha k}|^2\right)^2 - 2 |U_{\alpha 1}|^2 |U_{\alpha 2}|^2 . \tag{B5}$$

Combining Eqs.(B4) and (B5), we obtain the following expression for the survival probability,

$$P_{\nu_\alpha \to \nu_\alpha} = \left(1 - \sum_{k=3,4} |U_{\alpha k}|^2\right)^2 P_{\nu_\alpha \to \nu_\alpha}^{(1,2)} + \left(\sum_{k=3,4} |U_{\alpha k}|^2\right)^2 , \tag{B6}$$

where

$$P_{\nu_\alpha \to \nu_\alpha}^{(1,2)} = 1 - \frac{2 |U_{\alpha 2}|^2 |U_{\alpha 1}|^2}{\left(\displaystyle\sum_{j=1,2} |U_{\alpha j}|^2\right)^2}\left(1 - \cos\frac{\Delta m_{21}^2 L}{2p}\right) \tag{B7}$$

is the survival probability of ν_α due to its mixing with ν_1 and ν_2. In Section II we have used Eq.(B6).

Equation (B7) can be written in terms of a mixing angle θ_α defined by

$$\cos\theta_\alpha = \frac{|U_{\alpha 1}|}{\sqrt{\left(\displaystyle\sum_{j=1,2} |U_{\alpha j}|^2\right)^2}} \qquad\qquad \sin\theta_\alpha = \frac{|U_{\alpha 2}|}{\sqrt{\left(\displaystyle\sum_{j=1,2} |U_{\alpha j}|^2\right)^2}} . \tag{B8}$$

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Then the expression (B7) for the survival probability $P^{(1,2)}_{\nu_\alpha \to \nu_\alpha}$ takes the standard two-generation form,

$$P^{(1,2)}_{\nu_\alpha \to \nu_\alpha} = 1 - \frac{1}{2} \sin^2 2\theta_\alpha \left(1 - \cos \frac{\Delta m^2_{21} L}{2 \, p} \right) \tag{B9}$$

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FIGURES

FIG. 1. Exclusion regions in the $A_{\nu_\mu;\nu_e}$–Δm^2 plane for small $|U_{e1}|^2$ and $|U_{\mu1}|^2$ in the model with mixing of three neutrinos discussed in Section II. The regions excluded by the BNL E776 and KARMEN $\nu_\mu \to \nu_e$ appearance experiments are bounded by the dash-dotted and dash-dot-dotted curves, respectively. The dashed line represents the results of the Bugey experiment. The curve passing through the circles is obtained from the results of the Bugey, CDHS and CCFR84 experiments using Eq.(2.27). The curve passing through the triangles is obtained from the results of the Bugey, FNAL E531 and CCFR95 experiments using Eq.(2.29). The line passing through the squares bounds the region that will be explored by CHORUS and NOMAD. The region allowed by the LSND experiment is also shown as the shadowed region limited by the two solid curves.

FIG. 2. Exclusion regions in the $A_{\nu_e;\nu_\tau}$–Δm^2 for small $|U_{e1}|^2$ and large $|U_{\mu1}|^2$ in the model with mixing of three neutrinos discussed in Section II. The solid line represents the results of the FNAL E531 experiment. The dashed line represents the results of the Bugey experiment. The curve passing through the triangles is obtained from the results of the Bugey, CDHS and CCFR84 experiments using Eq.(2.34). The curve passing through the squares is obtained from the results of the CDHS, CCFR84 and BNL E776 experiments using Eq.(2.37). The curve passing through the circles is obtained from the results of the BNL E776, FNAL E531 and CCFR95 experiments using Eq.(2.38).

FIG. 3. Exclusion regions in the $A_{\nu_\mu;\nu_e}$–Δm^2 plane for small b_e and b_μ in the model with mixing of four neutrinos discussed in Section III. The regions excluded by the BNL E776 and KARMEN $\nu_\mu \to \nu_e$ appearance experiments are bounded by the dash-dotted and dash-dot-dotted curves, respectively. The dashed line represents the results of the Bugey experiment. The curve passing through the circles is obtained from the results of the Bugey, CDHS and CCFR84 experiments using Eq.(3.11).
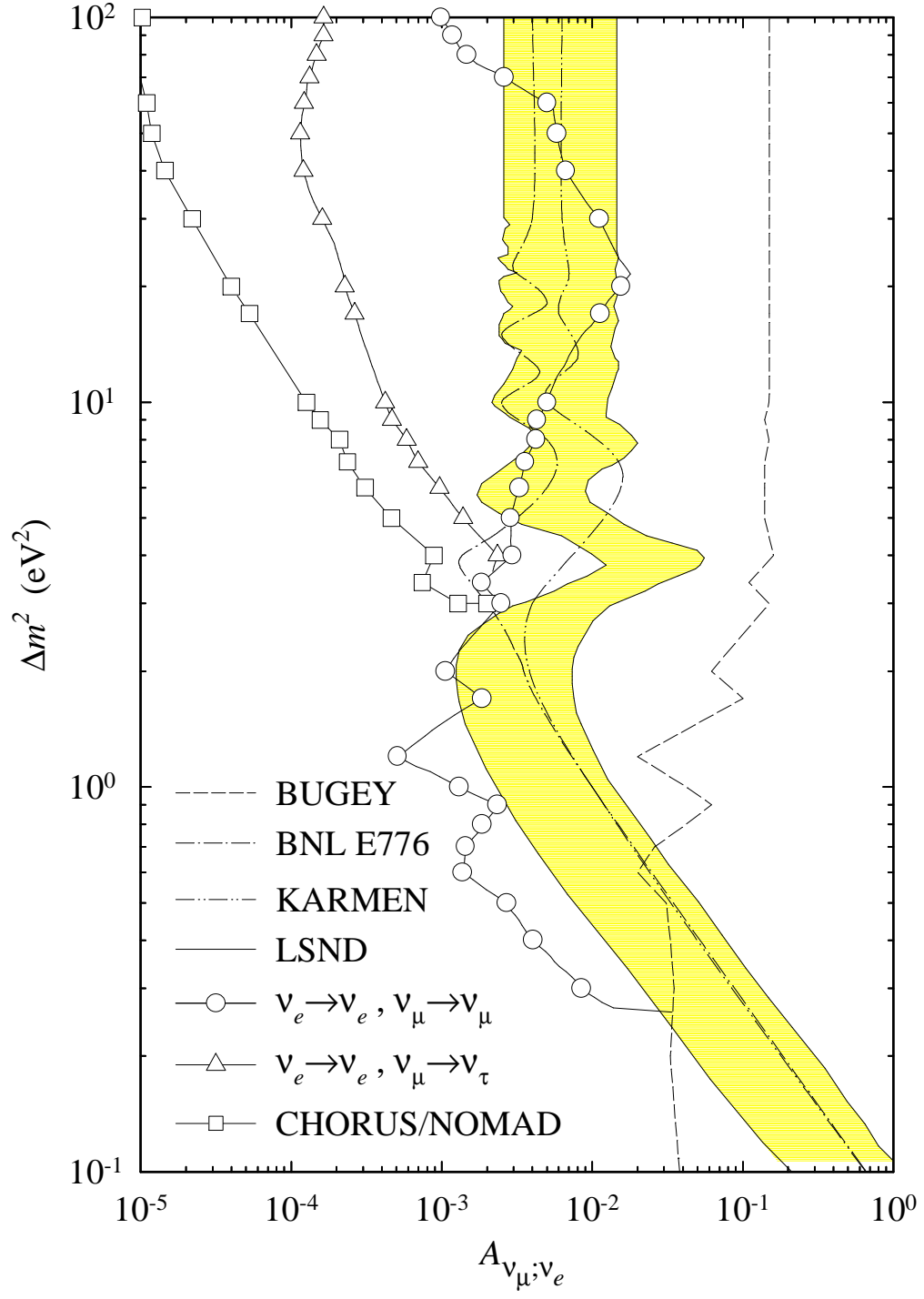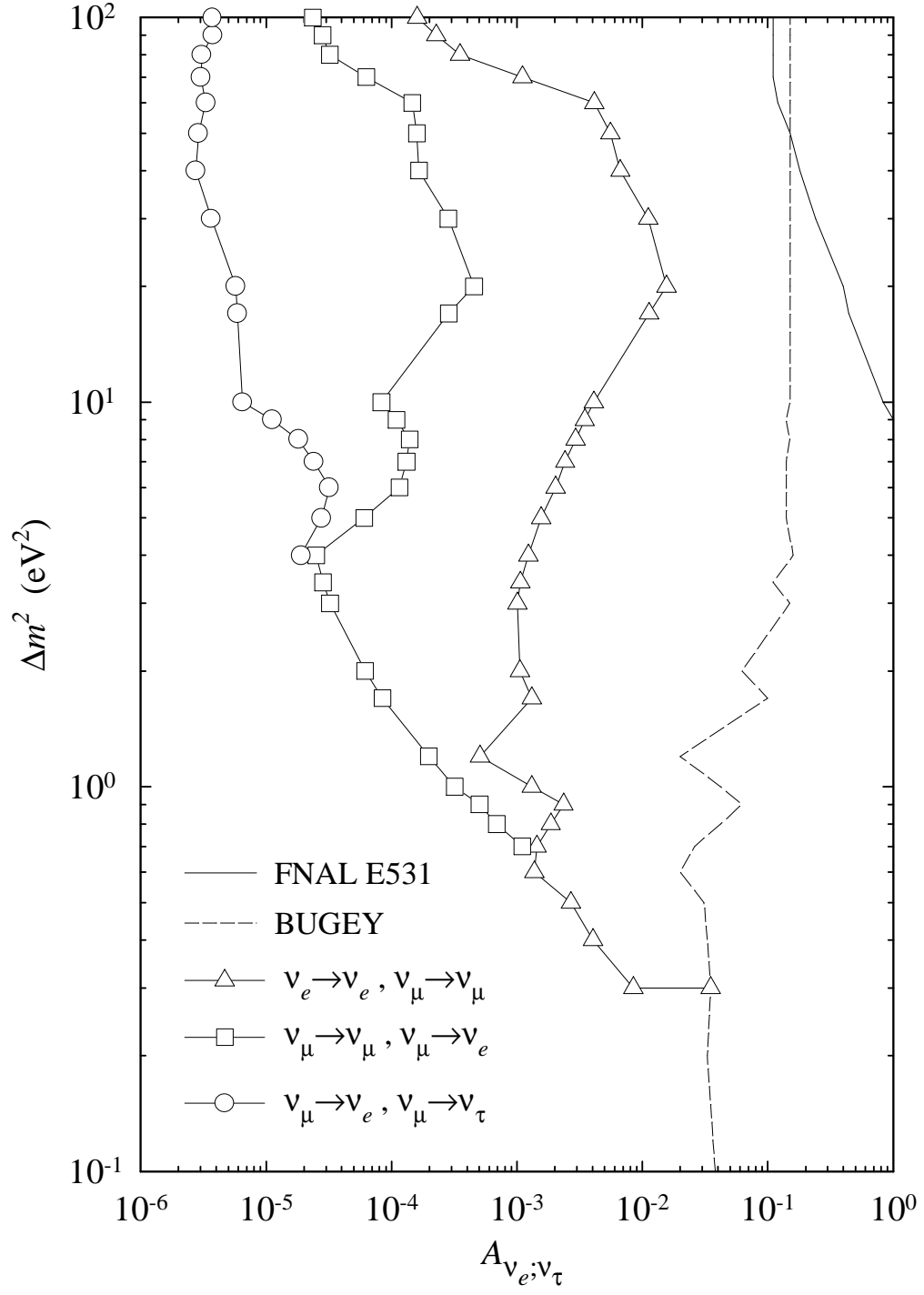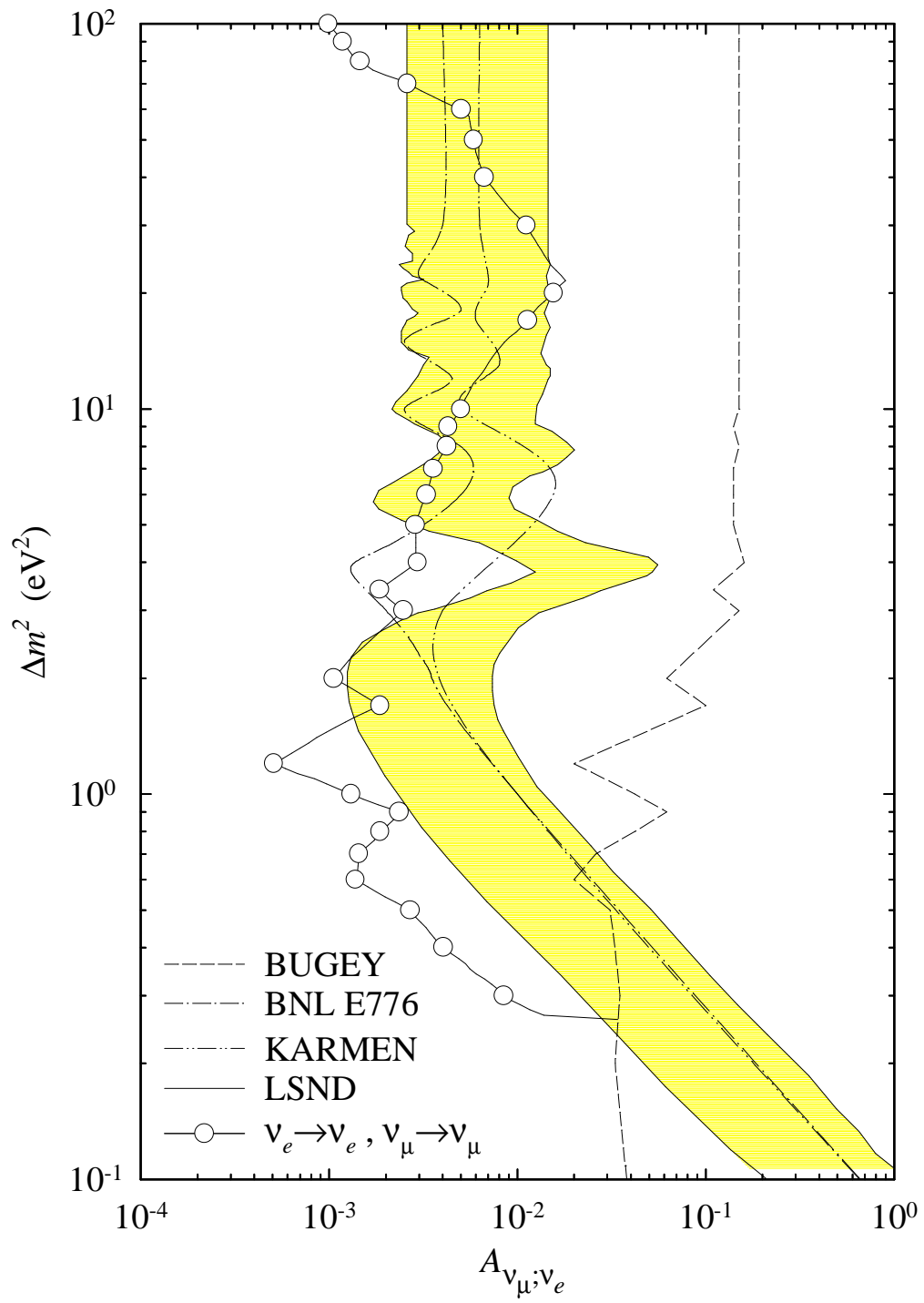
Figure 1

Figure 2

Figure 3